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APPLICATION OF THE NONPARAMETRIC GOODNESS-OF-FIT TESTS IN TESTING COMPOSITE HYPOTHESES*

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It is shown that in testing composite hypotheses the distribution laws of the statistics of goodness-of-fit tests are substantially affected by a set of factors: the form of the law observed; the type of parameter estimated and the number of estimated parameters; sometimes, a concrete parameter value; the method for estimating the parameters. Constructed approximations of the limiting statistic distributions of the nonparametric goodness-of-fit tests are recommended for application. They extend the region of correct usage of these tests for testing composite hypotheses.

INTRODUCTION

Testing for fit of an obtained experimental distribution and a theoretical one is one of the most common problems of statistical analysis in processing of experimental observation results. Applying the goodness-of-fit tests, one distinguishes testing for simple and composite hypotheses. A simple hypothesis tested has the form $H_0: F(x) = F(x, \theta)$, where $F(x, \theta)$ is the probability distribution function to which the observed sample is tested for fit, and θ is the known value of the parameter (either scalar or vector one). A composite tested hypothesis has the form $H_0: F(x) \in \{F(x, \theta),\ \theta \in \Theta\}$. In this case, the estimate of the distribution parameter $\hat{\theta}$ is calculated by the same sample by which the fit is tested.

While testing the fit by a sample we calculate the value S^* of statistic of the test used. To obtain the conclusion on accepting or rejecting the hypothesis H_0, we must know the conditional distribution $G(S \mid H_0)$ of statistic S under validity of hypothesis H_0. And if the probability

$$P\{S > S^*\} = \int_{S^*}^{\infty} g(s \mid H_0)\, ds$$

is sufficiently great, at least $P\{S > S^*\} > \alpha$, where $g(s \mid H_0)$ is the conditional density, and α is the prescribed significance level (the probability of a first-kind error – to reject the true hypothesis H_0), then it is usually considered that there are no grounds for rejecting the hypothesis H_0.

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The most commonly used goodness-of-fit tests, include nonparametric Kolmogorov tests and also Mises ω^2 and Ω^2 tests. The value

$$D_n = \sup_{|x|<\infty} |F_n(x) - F(x,\theta)|,$$

where $F_n(x)$ is the empirical distribution function, $F(x,\theta)$ is the theoretical distribution function, and n is the sample size, is used as a distance between the empirical and theoretical laws in Kolmogorov test. For testing hypotheses, one usually uses statistic of the form [1]

$$S_K = \frac{6nD_n + 1}{6\sqrt{n}}$$

Where $D_n = \max(D_n^+, D_n^-)$, $D_n^+ = \max\limits_{1 \le i \le n}\left\{\frac{i}{n} - F(x_i,\theta)\right\}$, $D_n^- = \max\limits_{1 \le i \le n}\left\{F(x_i,\theta) - \frac{i-1}{n}\right\}$, $x_1, x_2, \ldots, x_n$ are sample values in increasing order, and $F(x,\theta)$ is the function of the distribution law, fit to which is tested. The distribution of statistic S_K in testing the simple hypothesis in the limit obeys Kolmorogov law $K(S)$ [1].

In tests of the type of ω^2, the distance between the hypothetical and true distributions is considered in the quadratic metric

$$\int_{-\infty}^{\infty} \{E[F_n(x)] - F(x)\}^2 \psi(F(x))dF(x)$$

where $E[\cdot]$ is the mathematical expectation operator.

In choosing $\psi(t) \equiv 1$ in Mises ω^2 tests, one uses a statistic (Cramer – Mises – Smirnov statistic) of the form

$$S_\omega = n\omega_n^2 = \frac{1}{12n} + \sum_{i=1}^{n}\left\{F(x_i,\theta) - \frac{2i-1}{2n}\right\}^2.$$

In testing a simple hypothesis it obeys the distribution $a_1(S)$ [1].

In choosing $\psi(t) \equiv 1/t(1-t)$ in Mises Ω^2 tests, the statistic (Anderson – Darling statistic) has the form

$$S_\Omega = n\Omega_n^2 = -n - 2\sum_{i=1}^{n}\left\{\frac{2i-1}{2n}\ln F(x_i,\theta) + \left(1 - \frac{2i-1}{2n}\right)\ln(1 - F(x_i,\theta))\right\}.$$

In the limit, this statistic obeys the distribution $a_2(S)$ [1].

In the case of simple hypotheses, the limiting statistic distributions of the nonparametric Kolmogorov, Mises ω^2 and Ω^2 tests are known for a long time and do not depend on the kind of the distribution law observed and its parameters. These tests are said to be "distribution-free" tests. This advantage predetermines common applications of these tests.

1. **Losing the "distribution freeness" in testing composite hypotheses.** While testing the composite hypotheses, when the same sample is used to estimate the parameters of the observed law $F(x,\theta)$, the nonparametric goodness-of-fit tests lose the property of "distribution freeness". However, the nonparametric test power in testing the composite hypotheses for the same sample sizes is always much higher than that in testing simple ones. And whereas in testing the simple hypotheses the nonparametric Kolmogorov, ω^2 and Ω^2 Mises tests have a lower power compared with the χ^2-type tests provided that the latter use the asymptotically optimal grouping [2–5], in testing the composite hypotheses the nonparametric tests appear to be more powerful. To

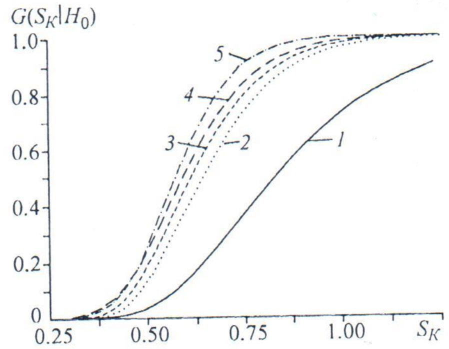

Fig. 1. Distribution functions $G(S_K \mid H_0)$ of statistic S_K of Kolmogorov test: *1* – for simple hypothesis testing; *2–5* – for calculating MLE of two parameters of Laplace, normal, Cauchy, and logistic distributions, respectively.

make use of their advantages, we must merely know the distribution $G(S \mid H_0)$ for the tested composite hypotheses.

The distinctions in the limiting distributions of the same statistics in testing simple and composite hypotheses are such significant that we cannot neglect them. Hence, many publications [6–8] warned against inaccurate application of the goodness-of-fit tests in testing composite hypotheses.

Paper [9] was the pioneer in investigating the limiting statistic distributions of the nonparametric goodness-of-fit tests in testing the composite hypotheses. The literature presents several approaches to using the nonparametric goodness-of-fit tests in the case of testing the composite hypotheses. When the size of a sample is large, it can be divided into two parts, and use one part for estimating the parameters and the other part for testing the fit [10]. In some particular cases, the limiting statistic distributions were analyzed analytically [11], the percent points of the distributions were constructed by statistical simulation [12–15]. For approximate calculation of the probabilities of "fit" of the form of $P\{S > S^*\}$ (achievable significance level), some authors constructed formulas that give sufficiently good approximations for small values of the corresponding probabilities [16–20]. Authors of [21–24] investigated the statistic distributions of the nonparametric goodness-of-fit tests and constructed models of these distributions by means of computer analysis of statistical regularities.

It has been found that in composite hypotheses tests, the conditional distribution law of the statistic $G(S \mid H_0)$ is affected by a number of factors determining the hypothesis complexity: the form of the observed law $F(x, \theta)$ corresponding to the true hypothesis H_0; the type of the parameter estimated and the number of parameters to be estimated; sometimes, it is a concrete value of the parameter (e.g., in the case of gamma-distribution); the method of parameter estimation.

For example, Fig. 1 illustrates $G(S \mid H_0)$ as a function of form of the observed law $F(x, \theta)$ corresponding to hypothesis H_0 for Kolmogorov test. Figure 2 shows distributions of the Mises ω^2 statistic for test of fit to Weibull distribution using various estimation methods: maximum likelihood estimates (MLE) and minimum distance (MD) estimates obtained by minimizing the value of the statistic used in the test.

Distributions of the goodness-of-fit statistics depend substantially on the parameter estimation method. Strictly speaking, each type of estimates for a concrete composite hypothesis tested is associated with its own limiting statistic distribution $G(S \mid H_0)$. Applying the nonparametric goodness-of-fit tests, one must consider the estimation method used. For the maximum likelihood method the statistic distributions $G(S \mid H_0)$ depend strongly on the law corresponding to hypothesis H_0. The scatter of the distributions $G(S \mid H_0)$ when using the MD estimates minimizing the test statistic is much less dependent on the law $F(x, \theta)$ corresponding to hypothesis H_0.

For the MD estimates minimizing the test statistic, the empirical distributions $G(S_n \mid H_0)$ corresponding to different hypotheses H_0 have the minimal scatter. Hence, we may speak about certain "distribution freeness" for the tests considered. If we use only this fact as a background, it would seem that only such estimation methods

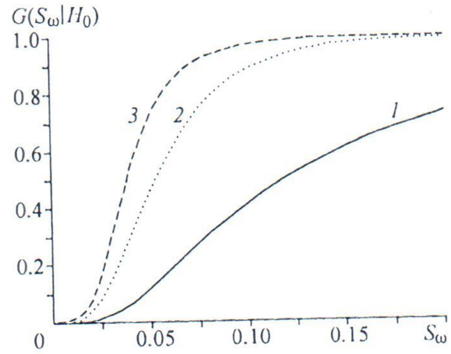

Fig. 2. Distribution functions $G(S_\omega \mid H_0)$ of statistic of the Mises ω^2 test for test of fit to Weibull distribution: *1* – for simple hypothesis testing; *2* – for calculating MLE of two parameters of distribution; *3* – for calculating MD estimates of two parameters.

must be applied for testing composite hypotheses. However, investigation of the power of the tests considered using different estimation methods has shown that for close alternatives the tests have the greatest power in the case of using MLE.

For small sample sizes the distributions $G(S_n \mid H_0)$ depend on n. However, we observe a substantial dependence of the statistic distribution on n only for small sample sizes. Investigation has shown, that for $n \geq 15$–20 the distributions $G(S_n \mid H_0)$ are sufficiently close to the limiting $G(S \mid H_0)$, and we may neglect the dependence on n.

2. **Construction of approximations for the limiting statistic distributions**. The limiting statistic distributions of the nonparametric tests and tables of the percent points constructed by the present moment are bounded by a rather narrow range of composite hypotheses.

The infinite number of random variables that can be met in practice cannot be described by a limited subset of models of distribution laws that are most frequently used to describe real observations. Any researcher can propose (construct) a parametric distribution model of his own for a concrete observed variable, i.e., such a model that describes most adequately this random variable, from his point of view. Upon estimating by the given sample the model parameters, it becomes necessary to test the hypothesis on adequacy of the sample observations and the constructed law by means of the goodness-of-fit tests. The next problem consists in knowledge of the limiting statistic distribution corresponding to the given composite hypothesis.

Construction of the limiting distribution by analytical methods is an extremely complicated problem. It is most suitable to use the method of computer analysis of statistical regularities. This method showed good results in simulating the test statistic distributions [21–24].

For this purpose, we must according to the law $F(x, \hat{\theta})$ simulate N samples of the same size n as the sample for which we must test hypothesis H_0: $F(x) \in \{F(x, \theta),\ \theta \in \Theta\}$, and then for each of N samples calculate estimates of the same law parameters and the value of statistic S of the corresponding goodness-of-fit test. As a result, we will obtain a sample of values of statistic $S_1, S_2, \ldots, S_N$ with the distribution law $G(S_n \mid H_0)$ for the hypotheses H_0. With this sample for a considerable number N we can construct a sufficiently smooth empirical function of distribution $G_N(S_n \mid H_0)$, that can be used in order to conclude whether we must accept hypothesis H_0. If it is necessary, using $G_N(S_n \mid H_0)$ we can construct an approximate analytical model approximating $G_N(S_n \mid H_0)$, and then, on the basis of this model, make the decision on the hypothesis tested.

Investigation has shown that a good analytical model for $G_N(S_n \mid H_0)$ is often represented by one of the following laws: log-normal, gamma distribution, *Su*-Johnson distribution, and *Sl*-Johnson distribution [23, 24]. At least, basing on the limited number of distribution laws, we can always construct a model in the form of a mixture of laws.

Implementation of such a procedure of computer analysis of statistic distributions contains neither difficulties of principal nor practical difficulties at present. The level of computing allows one to obtain quickly results of simulation, and an engineer who can write programs is able to implement the algorithm. In this paper we constructed models approximating the limiting statistic distributions for some composite hypotheses with the use of the MLE and MD estimates.

Table 1 contains a list of distributions relative to which we can test composite fit hypotheses using the constructed approximations of the limiting statistic laws. The statistic distribution models constructed by applying the method of computer analysis of statistical regularities are presented in Tables 2–7. Tables of the models of the limiting statistic distributions, including tables of percent points, for a wider class of tested composite hypotheses are listed on WEB-site [25].

Table 1

List of Distributions Corresponding to Tested Hypothesis H_0

Random variable distribution	Density function
Exponential	$\frac{1}{\theta_0} e^{-x/\theta_0}$
Seminormal	$\frac{2}{\theta_0\sqrt{2\pi}} e^{-x^2/2\theta_0^2}$
Rayleigh	$\frac{x}{\theta_0^2} e^{-x^2/2\theta_0^2}$
Maxwell	$\frac{2x^2}{\theta_0^3\sqrt{2\pi}} e^{-x^2/2\theta_0^2}$
Laplace	$\frac{1}{2\theta_0} e^{-\lvert x-\theta_1\rvert/\theta_0}$
Normal	$\frac{1}{\theta_0\sqrt{2\pi}} e^{-(x-\theta_1)^2/2\theta_0^2}$
Log-normal	$\frac{1}{x\theta_0\sqrt{2\pi}} e^{-(\ln x-\theta_1)^2/2\theta_0^2}$
Cauchy	$\frac{\theta_0}{\pi[\theta_0^2+(x-\theta_1)^2]}$
Logistic	$\frac{\pi}{\theta_0\sqrt{3}} \exp\left\{-\frac{\pi(x-\theta_1)}{\theta_0\sqrt{3}}\right\} \Big/ \left[1+\exp\left\{-\frac{\pi(x-\theta_1)}{\theta_0\sqrt{3}}\right\}\right]^2$
Extreme-value (maximum)	$\frac{1}{\theta_0} \exp\left\{-\frac{x-\theta_1}{\theta_0} - \exp\left(-\frac{x-\theta_1}{\theta_0}\right)\right\}$
Extreme-value (minimum)	$\frac{1}{\theta_0} \exp\left\{\frac{x-\theta_1}{\theta_0} - \exp\left(\frac{x-\theta_1}{\theta_0}\right)\right\}$
Weibull	$\frac{\theta_0 x^{\theta_0-1}}{\theta_1^{\theta_0}} \exp\left\{-\left(\frac{x}{\theta_1}\right)^{\theta_0}\right\}$
Gamma-distribution	$\frac{1}{\theta_1^{\theta_0}\Gamma(\theta_0)}(x-\theta_2)^{\theta_0-1} e^{-(x-\theta_2)/\theta_1}$

Table 2

Approximation of Limiting Kolmogorov Statistic Distributions Using the Maximum Likelihood Method

Random variable distribution	Estimation of		
	scale parameter	bias parameter	two parameters
Exponential	$\ln N(-0.3422, 0.2545)$	–	–
Seminormal	$\gamma(4.1332, 0.1076, 0.3205)$	–	–
Rayleigh	$\ln N(-0.3388, 0.2621)$	–	–
Maxwell	$\ln N(-0.3461, 0.2579)$	–	–
Laplace	$\gamma(3.7580, 0.1365, 0.3163)$	$\gamma(4.6474, 0.0870, 0.3091)$ $\ln N(-0.3690, 0.2499)$	$\gamma(4.4525, 0.0761, 0.3252)$ $\ln N(-0.4358, 0.2276)$
Normal	$\gamma(3.7460, 0.1385, 0.3142)$	$\ln N(-0.4172, 0.2272)$	$\gamma(4.9014, 0.0691, 0.2951)$ $\ln N(-0.4825, 0.2296)$
Log-normal	$\gamma(3.0622, 0.1577, 0.3547)$	$Su(-2.0328, 2.3642, 0.2622, 0.4072)$	$Su(-1.8093, 1.9041, 0.1861, 0.4174)$
Cauchy	$Su(-3.3278, 2.2529, 0.2185, 0.2858)$	$\gamma(4.8247, 0.0874, 0.2935)$	$\ln N(-0.5302, 0.2427)$
Logistic	$\gamma(3.2167, 0.1476, 0.3538)$	$Su(-2.8534, 3.0657, 0.2872, 0.3199)$	$\ln N(-0.5611, 0.2082)$
Extreme-value (maximum)	$\gamma(3.3841, 0.1439, 0.3509)$	$\gamma(4.1008, 0.0997, 0.3269)$	$\gamma(4.9738, 0.0660, 0.3049)$
Extreme-value (minimum)	$\gamma(3.3841, 0.1439, 0.3509)$	$\gamma(4.1008, 0.0997, 0.3269)$	$\gamma(4.9738, 0.0660, 0.3049)$
Weibull	$\gamma(3.3841, 0.1439, 0.3509)$**	$\gamma(4.1008, 0.0997, 0.3269)$*	$\gamma(4.9738, 0.0660, 0.3049)$

Note. ** – we estimated the Weibull distribution form parameter, * – the Weibull distribution scale parameter.

Table 3

Approximation of Limiting Distributions of Kolmogorov Minimum Statistic Using the MD Estimates Minimizing Statistic S_K

Random variable distribution	Estimation of		
	scale parameter	bias parameter	two parameters
Exponential	γ(4.4983, 0.0621, 0.2891)	–	–
Seminormal	γ(4.2884, 0.0705, 0.3072)	–	–
Rayleigh	γ(4.8579, 0.0639, 0.2900)	–	–
Maxwell	γ(5.3106, 0.0581, 0.2865)	–	–
Laplace	γ(3.0431, 0.1355, 0.3182)	γ(5.0103, 0.0602, 0.2968) ln*N*(–0.5358, 0.2122)	*Su*(–2.1079, 2.4629, 0.1661, 0.3340) ln*N*(–0.6970, 0.1952)
Normal	γ(3.2458, 0.1343, 0.3072)	ln*N*(–0.5469, 0.2152)	ln*N*(–0.7236, 0.1837)
Log-normal	γ(3.2458, 0.1343, 0.3072)	ln*N*(–0.5469, 0.2152)	ln*N*(–0.7236, 0.1837)
Cauchy	γ(3.4398, 0.1255, 0.3022)	ln*N*(–0.5182, 0.2268)	*Su*(–1.6929, 2.5234, 0.1892, 0.3607) ln*N*(–0.6946, 0.1938)
Logistic	*Su*(–2.6522, 1.8288, 0.1738, 0.3384) γ(3.6342, 0.1284, 0.2772)	*Su*(–3.8497, 3.2770, 0.2136, 0.2607) ln*N*(–0.5511, 0.2045)	ln*N*(–0.7389, 0.1771) *Su*(–2.5093, 3.1277, 0.1932, 0.3041)
Extreme-value (maximum)	γ(3.5424, 0.1203, 0.2975)	*Su*(–1.9028, 2.3972, 0.2227, 0.389)	*Su*(–1.3144, 2.2480, 0.1616, 0.3858) ln*N*(–0.7174, 0.1841)
Extreme-value (minimum)	γ(3.5424, 0.1203, 0.2975)	*Su*(–1.9028, 2.3972, 0.2227, 0.389)	*Su*(–1.3144, 2.2480, 0.1616, 0.3858) ln*N*(–0.7174, 0.1841)
Weibull	γ(3.5424, 0.1203, 0.2975)**	*Su*(–1.9028, 2.3972, 0.2227, 0.389)*	*Su*(–1.3144, 2.2480, 0.1616, 0.3858) ln*N*(–0.7174, 0.1841)

Note. ** – we estimated the Weibull distribution form parameter, * – the Weibull distribution scale parameter.

Table 4

Approximation of Limiting Distributions of Mises ω^2 Statistic Using the Maximum Likelihood Method

Random variable distribution	Estimation of		
	scale parameter	bias parameter	two parameters
Exponential	*Su*(–1.8734, 1.2118, 0.0223, 0.0240)	–	–
Seminormal	*Sl*(0.9735, 1.1966, 0.1531, 0.0116)	–	–
Rayleigh	*Su*(–1.5302, 1.0371, 0.0202, 0.0299)	–	–
Maxwell	*Su*(–2.0089, 1.2557, 0.0213, 0.0213)	–	–
Laplace	*Sl*(0.9719, 0.9805, 0.2347, 0.0139)	*Su*(–2.0821, 1.2979, 0.0196, 0.0200)	*Su*(–1.6085, 1.2139, 0.0171, 0.0247)
Normal	*Su*(–2.2550, 0.9569, 0.0152, 0.0212)	ln*N*(–2.7536, 0.5610)	ln*N*(–2.9794, 0.5330)
Log-normal	*Sl*(1.0669, 1.0010, 0.2537, 0.0144)	ln*N*(–2.7271, 0.6092)	*Su*(–1.6292, 1.1541, 0.0144, 0.0234)
Cauchy	*Sl*(1.0086, 1.0539, 0.2282, 0.0064)	*Sl*(1.1230, 1.2964, 0.1383, 0.0105)	*Sl*(1.2420, 1.2833, 0.1135, 0.0064)
Logistic	*Sl*(0.9982, 1.0287, 0.2303, 0.0126)	*Sl*(1.3982, 1.3804, 0.1205, 0.0102)	ln*N*(–3.1416, 0.4989)
Extreme-value (maximum)	*Sl*(1.0056, 1.0452, 0.2296, 0.0137)	ln*N*(–2.5818, 0.6410)	ln*N*(–2.9541, 0.5379)
Extreme-value (minimum)	*Sl*(1.0056, 1.0452, 0.2296, 0.0137)	ln*N*(–2.5818, 0.6410)	ln*N*(–2.9541, 0.5379)
Weibull	*Sl*(1.0056, 1.0452, 0.2296, 0.0137)**	ln*N*(–2.5818, 0.6410)*	ln*N*(–2.9541, 0.5379)

Note. ** – we estimated the Weibull distribution form parameter, * – the Weibull distribution scale parameter.

Table 5

Approximation of Limiting Distributions of Minimum Mises ω^2 Statistic Using the MD Estimates Minimizing Statistic S_{ω}

Random variable distribution	Estimation of		
	scale parameter	bias parameter	two parameters
Exponential	$Su(-1.9324, 1.1610, 0.0134, 0.0203)$	–	–
Seminormal	$Su(-1.5024, 1.0991, 0.0173, 0.0256)$	–	–
Rayleigh	$Su(-1.4705, 1.1006, 0.0164, 0.0259)$	–	–
Maxwell	$Su(-1.7706, 1.2978, 0.0188, 0.0220)$	–	–
Laplace	$Sl(1.0117, 0.9485, 0.2162, 0.0137)$	$\ln N(-2.8601, 0.5471)$	$\ln N(-3.2853, 0.4666)$
Normal	$Sl(1.0477, 0.9883, 0.2356, 0.0112)$	$\ln N(-2.8649, 0.5668)$	$\ln N(-3.2715, 0.4645)$
Log-normal	$Sl(1.0477, 0.9883, 0.2356, 0.0112)$	$\ln N(-2.8649, 0.5668)$	$\ln N(-3.2715, 0.4645)$
Cauchy	$Sl(1.2759, 1.0437, 0.2825, 0.0089)$	$\ln N(-2.8577, 0.5739)$	$\ln N(-3.2603, 0.4874)$
Logistic	$Sl(1.0898, 1.0225, 0.2399, 0.0096)$	$\ln N(-2.8831, 0.5367)$	$\ln N(-3.2915, 0.4592)$
Extreme-value (maximum)	$Sl(1.0771, 1.0388, 0.2065, 0.0109)$	$Su(-1.5348, 1.1226, 0.0166, 0.0252)$	$Su(-1.5326, 1.4446, 0.0147, 0.0188)$ $\ln N(-3.2627, 0.4680)$
Extreme-value (minimum)	$Sl(1.0771, 1.0388, 0.2065, 0.0109)$	$Su(-1.5348, 1.1226, 0.0166, 0.0252)$	$Su(-1.5326, 1.4446, 0.0147, 0.0188)$ $\ln N(-3.2627, 0.4680)$
Weibull	$Sl(1.0771, 1.0388, 0.2065, 0.0109)$**	$Su(-1.5348, 1.1226, 0.0166, 0.0252)$*	$Su(-1.5326, 1.4446, 0.0147, 0.0188)$ $\ln N(-3.2627, 0.4680)$

Note. ** – we estimated the Weibull distribution form parameter, * – the Weibull distribution scale parameter.

Table 6

Approximation of Limiting Distributions of Mises Ω^2 Statistic Using the Maximum Likelihood Method

Random variable distribution	Estimation of		
	scale parameter	bias parameter	two parameters
Exponential	*Su*(−2.8653, 1.4220, 0.1050, 0.1128)	–	–
Seminormal	*Su*(−2.5603, 1.3116, 0.1147, 0.1330)	–	–
Rayleigh	*Su*(−2.5610, 1.4003, 0.1174, 0.1337)	–	–
Maxwell	*Su*(−2.6064, 1.4426, 0.1190, 0.1285)	–	–
Laplace	*Sl*(0.3148, 1.0999, 0.6901, 0.1093)	*Su*(−2.5528, 1.4006, 0.1216, 0.1358)	*Su*(−2.8942, 1.4897, 0.0846, 0.1131)
Normal	*Su*(−2.3507, 1.0531, 0.1012, 0.1595)	*Su*(−3.1202, 1.5233, 0.0874, 0.1087)	*Su*(−2.7057, 1.7154, 0.1043, 0.0925)
Log-normal	*Su*(−2.4168, 1.1296, 0,1151, 0.1560)	ln*N*(−0.8052, 0.5123)	*Su*(−2.3966, 1.5967, 0.1012, 0.1179)
Cauchy	*Su*(−2.4935, 1.0789, 0.0923, 0.1458)	*Su*(−2.8420, 1.3528, 0.1010, 0.1221)	*Su*(−2.3195, 1.1812, 0.0769, 0.1217)
Logistic	*Sl*(0.3065, 1.1628, 0.7002, 0.0930)	*Su*(−3.5408, 1.6041, 0.0773, 0.0829)	ln*N*(−1.1452, 0.4426)
Extreme-value (maximum)	*Su*(−2.5427, 1.1057, 0.0960, 0.1569)	*Su*(−2.5550, 1.3714, 0.1152, 0.1289)	*Su*(−2.4622, 1.6473, 0.1075, 0.1149)
Extreme-value (minimum)	*Su*(−2.5427, 1.1057, 0.0960, 0.1569)	*Su*(−2.5550, 1.3714, 0.1152, 0.1289)	*Su*(−2.4622, 1.6473, 0.1075, 0.1149)
Weibull	*Su*(−2.5427, 1.1057, 0.0960, 0.1569)**	*Su*(−2.5550, 1.3714, 0.1152, 0.1289)*	*Su*(−2.4622, 1.6473, 0.1075, 0.1149)

Note. ** – we estimated the Weibull distribution form parameter, * – the Weibull distribution scale parameter.

Table 7

Approximation of Limiting Distributions of Minimum Mises Ω^2 Statistic Using the MD Estimates Minimizing Statistic S_Ω

Random variable distribution	Estimation of		
	scale parameter	bias parameter	two parameters
Exponential	*Su*(−2.6741, 1.4068, 0.0958, 0.1230)	–	–
Seminormal	*Su*(−2.6752, 1.3763, 0.0952, 0.1280)	–	–
Rayleigh	*Su*(−2.2734, 1.3473, 0.1101, 0.1496)	–	–
Maxwell	*Su*(−2.2759, 1.3988, 0.1171, 0.1514)	–	–
Laplace	*Su*(−2.3884, 1.0811, 0.0948, 0.1548)	*Su*(−2.7267, 1.4972, 0.1044, 0.1239)	*Su*(−2.4334, 1.6104, 0.0902, 0.1123)
Normal	*Su*(−2.4180, 1.0702, 0.0957, 0.1464)	*Su*(−2.7639, 1.5393, 0.1102, 0.1115)	*Su*(−2.5746, 1.7505, 0.0979, 0.1043) ln*N*(−1.1651, 0.4271)
Log-normal	*Su*(−2.4180, 1.0702, 0.0957, 0.1464)	*Su*(−2.7639, 1.5393, 0.1102, 0.1115)	*Su*(−2.5746, 1.7505, 0.0979, 0.1043) ln*N*(−1.1651, 0.4271)
Cauchy	*Su*(−2.5043, 1.1355, 0.1035, 0.1384)	*Su*(−2.7029, 1.5179, 0.1188, 0.1100)	*Su*(−2.1046, 1.4364, 0.0929, 0.1301) ln*N*(−1.1043, 0.4692)
Logistic	*Sl*(0.3223, 1.1159, 0.6836, 0.0953) *Su*(−2.3007, 1.0135, 0.0906, 0.1593)	*Su*(−2.6212, 1.4318, 0.0932, 0.1370)	*Su*(−3.0152, 1.7751, 0.0800, 0.0898)
Extreme-value (maximum)	*Su*(−2.4454, 1.1083, 0.0968, 0.1459)	*Su*(−2.6557, 1.4282, 0.1024, 0.1254)	*Su*(−2.1580, 1.5446, 0.0941, 0.1279)
Extreme-value (minimum)	*Su*(−2.4454, 1.1083, 0.0968, 0.1459)	*Su*(−2.6557, 1.4282, 0.1024, 0.1254)	*Su*(−2.1580, 1.5446, 0.0941, 0.1279)
Weibull	*Su*(−2.4454, 1.1083, 0.0968, 0.1459)**	*Su*(−2.6557, 1.4282, 0.1024, 0.1254)*	*Su*(−2.1580, 1.5446, 0.0941, 0.1279)

Note. **– we estimated the Weibull distribution form parameter, * – the Weibull distribution scale parameter.

In tables 2–7, that contain the distributions $G(S \mid H_0)$ recommended for testing composite hypotheses, $\ln N(\theta_1, \theta_2)$ denotes the log-normal distribution with the density function

$$f(x) = \frac{1}{x\theta_0\sqrt{2\pi}} e^{-(\ln x - \theta_1)^2/2\theta_0^2},$$

$\gamma(\theta_0, \theta_1, \theta_2)$ denotes the gamma-distribution with the density function

$$f(x) = \frac{1}{\theta_1^{\theta_0}\Gamma(\theta_0)}(x-\theta_2)^{\theta_0 - 1} e^{-(x-\theta_2)/\theta_1},$$

$Sl(\theta_0, \theta_1, \theta_2, \theta_3)$ denotes the Sl-Johnson distribution with the density function

$$f(x) = \frac{\theta_1}{(x-\theta_3)} \exp\left\{-\frac{1}{2}\left[\theta_0 + \theta_1 \ln \frac{x-\theta_3}{\theta_2}\right]^2\right\},$$

and $Su(\theta_0, \theta_1, \theta_2, \theta_3)$ is the Su-Johnson distribution with the density function

$$f(x) = \frac{\theta_1}{\sqrt{2\pi}\sqrt{(x-\theta_3)^2 + \theta_2^2}} \exp\left\{-\frac{1}{2}\left[\theta_0 + \theta_1 \ln\left\{\frac{x-\theta_3}{\theta_2} + \sqrt{\left(\frac{x-\theta_3}{\theta_2}\right)^2 + 1}\right\}\right]^2\right\}.$$

As an example, we will show how strong is the change in the probability $P\{S > S^*\}$ for the same value of statistic in the case of simple and composite hypotheses. For illustration, we will use the Mises ω^2 test.

E x a m p l e. Let we test a hypotheses on fit to the Weibull distribution and the calculated statistic value $S_\omega^* = 0.14$. Hence, in the case of a simple hypotheses on the basis of the distribution $a_1(S)$ [1] we find that $P\{S_\omega > 0.14\} = 0.4215$. If using the sample we calculated MLE of two distribution parameters, then a good approximation of the limiting distribution (see Table 4) is the log-normal distribution $\ln N(-2.9541, 0.5379)$, and the corresponding probability $P\{S_\omega > 0.14\} = 0.0331$. For MD estimates in a similar situation, the most suitable model (see Table 5) is the Su-Johnson distribution $Su(-1.5326, 1.4446, 0.0147, 0.0188)$ according to which $P\{S_\omega > 0.14\} = 0.0058$.

CONCLUSION

To test composite hypotheses and choose (or construct) statistic distributions $G(S \mid H_0)$ of the goodness-of-fit tests, one should take into account all factors that affect the statistic distribution law: the form of the law observed; the type of the parameter estimated and the number of the parameters; sometimes, a concrete parameter value; the method of parameter estimation.

The constructed approximations of the limiting statistic distributions of the nonparametric goodness-of-fit tests extend the region of correct application of these tests and may be recommended to a wide range of researchers. The tested method for simulation of statistic distributions may be recommended for construction of statistical regularities when it is impossible to solve the problem analytically.

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