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# GENERAL PROBLEMS OF METROLOGY AND MEASUREMENT TECHNIQUE

## APPLICATION AND POWER OF CRITERIA FOR TESTING THE HOMOGENEITY OF VARIANCES. PART I. PARAMETRIC CRITERIA

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*A comparative analysis is made of the power of classical (Fisher, Bartlett, Cochran, Hartley, and Levene) tests of variance homogeneity. The distributions of the statistics of the tests are studied when the assumption that the sample obeys a normal law breaks down.*

***Key words:*** *tests of homogeneity of variances, Fisher, Bartlett, Cochran, Hartley, and Levene tests, power of tests.*

Tests of hypotheses of sample homogeneity are very frequently used in various applications of statistical analysis. This may involve testing hypothesis of homogeneity in the distribution laws corresponding samples being analyzed or hypotheses of homogeneity in the mathematical expectations or variances. The most complete conclusions are reached in the first case, but researchers may be more interested in possible deviations in the average values of samples or differences in the scatter of the data.

The application of the Smirnov and Lehmann–Rosenblatt nonparametric tests of variance has been discussed and their power analyzed in Ref. 1. The stability of the classical tests of the homogeneity of average values has been examined [2] with breakdown of the assumption that the random samples being analyzed adhere to a normal law, and a comparative analysis has been made of the power of the various tests, including nonparametric ones.

When classical tests of variance homogeneity are used, the question always remains of how correct the results are in a given specific situation. The problem is that one of the basic assumptions in constructing these tests is that the observed random quantities (measurement errors) are normally distributed. The parametric tests of variance homogeneity are extremely sensitive to the smallest deviation from a normal law for the observed random quantities. If this assumption is violated, the conditional distributions of the statistics of tests for the correctness of this hypothesis will, as a rule, change substantially. Since the errors of measurement instrumentation or the quantities observed in various applications are by no means always subject to a normal law, using the classical methods under these conditions may lead to incorrect conclusions.

In this regard, it is useful to study the behavior of tests for variance homogeneity (characteristics of spread) for certain deviations from normal in the distribution of the measurement results (controlled index) and the validity of using the classical apparatus for hypothesis testing. As yet, there is no clear answer as to how the power of the different parametric tests correlates with specific competing hypotheses and to what extent nonparametric tests of hypotheses of equality in dispersion characteristics (scaling parameters) are inferior to them in power.

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The following is a continuation of a study of the stability of tests of the equality of variance [3, 4]. The classical tests of Bartlett [5], Cochran [6], Fisher, Hartley [7], and Levene [8] are compared and the nonparametric (rank) tests of Ansari–Bradley [9], Mood [10], Siegel–Tukey [11], Capon [12], and Klotz [13] are examined.

The purpose of this work is, first of all, to study the distributions of the statistics of these tests for observed random samples that deviate from normal, second, to make a comparative analysis of the power of the tests with respect to specific competing hypotheses, third, to examine the feasibility of using the classical tests when the assumption of normally distributed random variables does not hold, and, fourth, to develop recommendations for the use of the tests under the conditions of actual applications.

The hypothesis of constant variances of $m$ samples and the competing hypothesis have the form

$$H_0 : \sigma_1^2 = \sigma_2^2 = ... = \sigma_m^2 ;$$

$$H_1 : \sigma_{i_1}^2 \neq \sigma_{i_2}^2 ,$$

where the inequality holds for at least one pair of indices $i_1$, $i_2$.

A study of the distributions of the statistics, the construction of the distributions of percentiles for these distributions, and an estimate of the power of the criteria with respect to various competing hypotheses have all been done using a method of statistical modelling (simulation) [14] in the framework of the Windows Controlled Interval Statistics (ISW) program system based on Ref. 15. The number of simulated samples of the statistics being studied was $N = 10^6$. For these values of $N$, the difference between the true distribution of the statistics and the simulated empirical distribution was less than $10^{-3}$ in absolute value.

The distributions of the statistics were studied for different observed distribution laws, in particular, in the case where the simulated samples belong to a family with the density

$$De(\theta_0) = f(x; \theta_0, \theta_1, \theta_2) = \frac{\theta_0}{2\theta_1 \Gamma(1/\theta_0)} \exp\left( -\left( \frac{|x - \theta_2|}{\theta_1} \right)^{\theta_0} \right) \quad (1)$$

for different values of the shape parameter $\theta_0$. This family can serve as a good model for the distributions of the errors of measurement systems. The distribution $De(\theta_0)$ includes the Laplace ($\theta_0 = 1$) and normal ($\theta_0 = 2$) distributions as special cases. The (1) family can be used to specify symmetric distributions which differ to some extent from normal: the smaller the shape parameter is, the "heavier" the tail of the distribution $De(\theta_0)$ will be, and *vice versa*.

For comparative analysis of the power of the tests, we have examined the following competing hypotheses: $\mathrm{H}_1 : \sigma_m = 1.1\sigma_0$; $H_2 : \sigma_m = 1.2\sigma_0$; $H_3 : \sigma_m = 1.5\sigma_0$. Thus, the competing hypothesis corresponds to a situation in which $(m - 1)$st sample corresponds to a distribution with some $\sigma = \sigma_0$, while one of the samples, e.g., that with the number $m$, has a different variance. The hypothesis to be tested corresponds to the situation $\mathrm{H}_0 : \sigma_1^2 = \sigma_2^2 = ... = \sigma_m^2 = \sigma_0^2$.

**The Bartlett test.** The statistic for this test is calculated using [16]

$$\chi^2 = M \left[ 1 + \frac{1}{3(m-1)} \left( \sum_{i=1}^{m} \frac{1}{\nu_i} - \frac{1}{N} \right) \right]^{-1} , \quad (2)$$

where

$$M = N \ln \left( \frac{1}{N} \sum_{i=1}^{m} \nu_i S_i^2 \right) - \sum_{i=1}^{m} \nu_i \ln S_i^2 ;$$

$\nu_i = n_i$ if the mathematical expectation is known and $\nu_i = n_i - 1$ if it is not; $m$ and $n_i$ are the number of samples and their sizes;

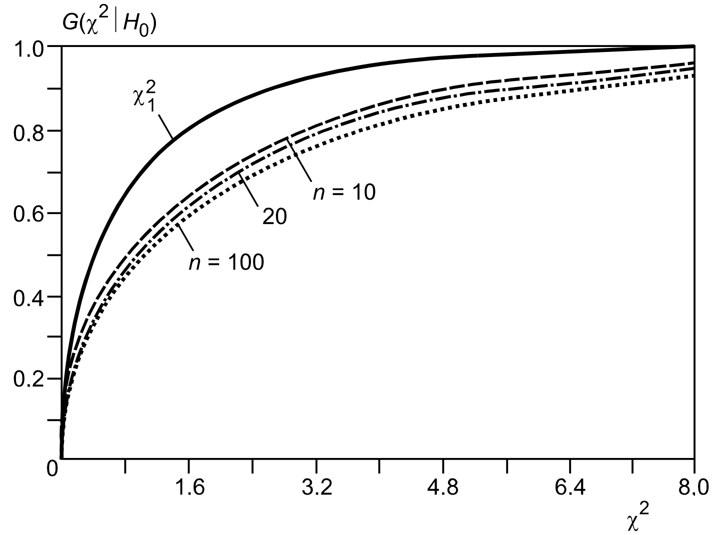


Fig. 1. Distribution functions of the statistic for the Bartlett test with different sizes of samples obeying a Laplace distribution with $m = 2$.

$$N = \sum_{i=1}^{m} \nu_i ;$$

and $S_i^2$ are the estimated sample variances.

When the mathematical expectation is unknown,

$$S_i^2 = \frac{1}{n_i - 1} \sum_{j=1}^{n_i} (X_{ji} - \overline{X}_i)^2 ,$$

where $X_{ij}$ is the $j$th observation in the $i$th sample, and

$$\overline{X}_i = \frac{1}{n_i} \sum_{j=1}^{n_i} X_{ji} .$$

If the hypothesis $H_0$ is true, then all the $\nu_i > 3$ and the samples are extracted from a normal general ensemble and the (2) statistic roughly follows a $\chi^2_{m-1}$ distribution. If the calculated value of the statistic $\chi^{2*} > \chi^2_{1-\alpha,\, m-1}$, then the test hypothesis is rejected at a specified significance level $\alpha$.

For normally distributed data, the distribution of the (2) statistic is essentially independent of changes in the sizes of the samples [3]. A known limiting distribution of the statistic and the possibility of using it for small sample sizes are serious advantages of the Bartlett test.

When the distribution of the observed index deviates from normal, the distribution $G(\chi^2 | H_0)$ of the (2) statistic comes to depend on the sample size and differs from $\chi^2_{m-1}$. This is illustrated by Fig. 1, which shows the distributions of the statistic for different sizes of two comparison samples that obey a Laplace distribution (the (1) family with the parameter $\theta_0 = 1$), together with the limiting $\chi^2_1$ distribution for the statistic (for the classical situation following a normal distribution).

The distributions of the (2) statistic are very sensitive to deviations of the observed distribution from normal. Figure 2 shows how this distribution changes for measurement data which conform to the (1) family with different values of the shape parameter.

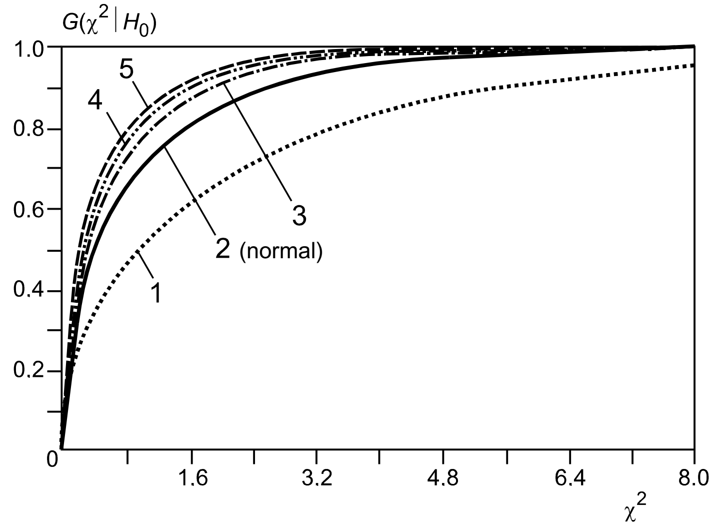


Fig. 2. Distribution functions for the Bartlett test statistic for the (1) family with different values of the shape parameter $\theta_0$ with $n = 20$ and $m = 2$.

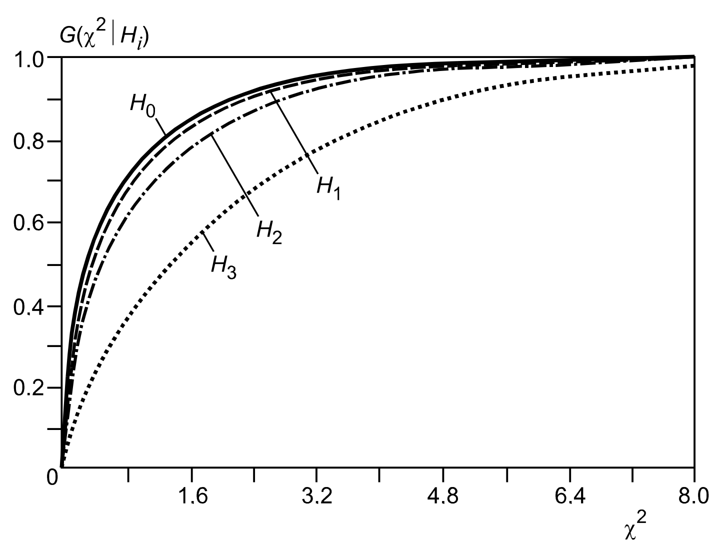


Fig. 3. Conditional distributions of the Bartlett test statistic with $m = 2$ and $n = 10$; the samples obey family (3) with shape parameter $\theta_0 = 3$.

Figure 3 shows plots of the distribution of the (2) statistic with true competing hypotheses $H_1 : \sigma_m = 1.1\sigma_0$; $H_2 : \sigma_m = 1.2\sigma_0$; $H_3 : \sigma_m = 1.5\sigma_0$, when the samples obey a distribution from the (1) family with shape parameter $\theta_0 = 3$.

**The Cochran test.** When all the $n_i$ are the same, i.e., $n_1 = n_2 = ... = n_m = n$, the simpler Cochran test [6] can be used with a statistic defined as

$$Q = S_{\max}^2 / (S_1^2 + S_2^2 + ... + S_m^2), \tag{3}$$

where $S_{\max}^2 = \max(S_1^2, S_2^2, ..., S_m^2)$; $m$ is the number of independent estimates of the variance (number of sample); and $S_i^2$ are the estimated sample variances.

The distributions of the Cochran test depend strongly on the size of the observed samples. These distributions are unknown, so the handbooks only give tables of percentiles for a limited number of values of $n$ for use in hypothesis testing. The null hypothesis is rejected for large values of the statistic.

The way the distributions $G(Q \mid H_0)$ of the (3) statistic for the Cochran test with a true test hypothesis $H_0$ depend on the observed sample distribution is analogous to that for the Bartlett distribution [3].

**The Hartley test**, like the Cochran test, is used for samples with equal sizes. Its statistic has the form [7]

$$F = S_{\max}^2 / S_{\min}^2, \tag{4}$$

where $S_{\min}^2 = \min(S_1^2, S_2^2, ..., S_m^2)$.

The degrees of freedom for the distribution of the statistic are the numbers $\nu_1 = m$ and $\nu_2 = n - 1$, which implies that the distributions of the statistic depend significantly on the sample size. The test hypothesis is rejected for large values of the statistic. Only tables of percentiles are given in the literature for the (4) statistic. The way the distributions $G(F \mid H_0)$ of the statistic for the Hartley test with a true test hypothesis $H_0$ depend on the observed sample distribution is analogous to that for the Bartlett and Cochran distributions [3].

**The Levene test.** The statistic for this test is calculated using the formula [8]

$$W = \frac{N - m}{m - 1} \frac{\sum_{i=1}^{m} n_i (\bar{Z}_{i\bullet} - \bar{Z}_{\bullet\bullet})^2}{\sum_{i=1}^{m} \sum_{j=1}^{n_i} (\bar{Z}_{ij} - \bar{Z}_{i\bullet})^2}, \tag{5}$$

where $m$ is the number of samples; $n_i$ is the size of the $i$th sample;

$$N = \sum_{i=1}^{m} n_i ;$$

$X_{ij}$ is the $j$th observation in the $i$th sample; $Z_{ij} = |X_{ij} - \bar{X}_{i\bullet}|$, with $\bar{X}_{i\bullet}$ being the average in the $i$th sample; and $\bar{Z}_{i\bullet}$, $\bar{Z}_{\bullet\bullet}$ are the respective averages of $Z_{ij}$ in the $i$th sample and over all the samples.

In many descriptions of this test, e.g., [17], it is shown that in the case where the samples obey a normal law and $H_0$ is true, this statistic obeys a Fisher distribution $F_{\nu_1,\nu_2}$ with $\nu_1 = m - 1$, $\nu_2 = N - m$ degrees of freedom. In fact, for sample sizes of 10–20, the elements of the distribution of the (5) statistic differ substantially from $F_{\nu_1,\nu_2}$, and this must be taken into account when using the test. The validity of this statement follows from the definition of the $Z_{ij}$, which do not follow a normal law in any case, so that Eq. (5) cannot obey a $F_{\nu_1,\nu_2}$ distribution. In this regard, the percentiles of the distribution have been found by statistical modelling [18]. It was found, however, that even for sample sizes $n_i \geq 40$ the maximum deviation of the distribution of the statistic from a Fisher distribution was no more than 0.005 (in the case where the samples being analyzed adhered to a normal distribution). The test hypothesis is rejected for large values of the statistic.

The behavior of the conditional distribution $G(W \mid H_0)$ of the statistic for the Levene test when the test hypothesis $H_0$ is true is illustrated in Fig. 4 as a function of the form of the distribution corresponding to the observed samples. This comparison shows that the Levene test is less sensitive to deviations from normal in the samples being analyzed (this is evident on comparison with Fig. 2.)

In the original Levene test, only the use of the sample averages was envisioned. It has been proposed [19] that the estimates of the average in a statistic of the form (5) should be sample median and truncated average ($Z_{ij} = |X_{ij} - \tilde{X}_{i\bullet}|$, where $\tilde{X}_{i\bullet}$ is the median in the $i$th sample; $Z_{ij} = |X_{ij} - \bar{X}'_{i\bullet}|$, where $\bar{X}'_{i\bullet}$ is the truncated average in the $i$th sample). It is assumed that in these cases the test becomes even more stable to failure of the assumption of a normal distribution.

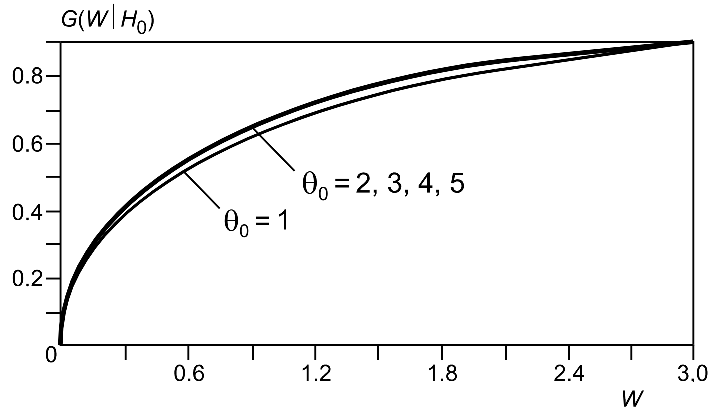


Fig. 4. Distribution functions of the Levene test statistic for distributions from the (1) family with different shape parameters $\theta_0$ and $n = 20$ and $m = 2$.

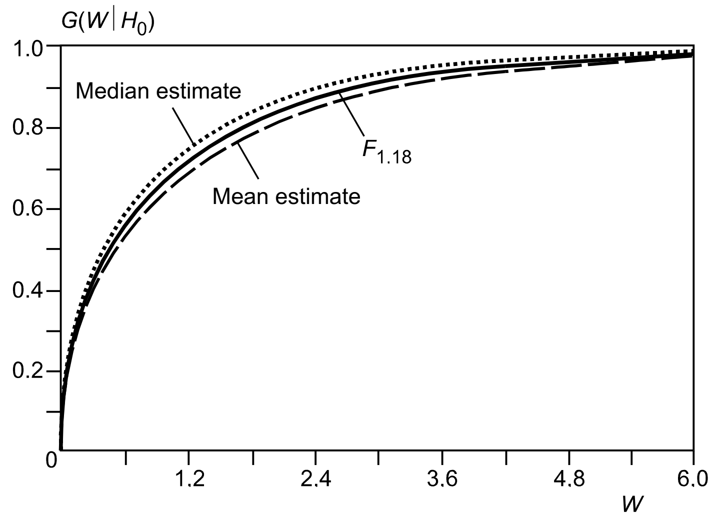


Fig. 5. Distribution functions of the Levene test statistic for normal-law samples with $n = 10$ and $m = 2$, along with the $F_{1.18}$ Fisher distribution.

Our studies, however, have clearly not confirmed this: the differences between the distributions $G(W|H_0)$ when the analyzed samples adhere to a normal law and to a Laplace law are of the same order of magnitude when either the ordinary mean value or, for example, the median is used in Eq. (5). Here it should be kept in mind that in the latter case, under the assumption of normal law behavior, this distribution differs substantially from the distribution using the ordinary mean and differs even more from a Fisher distribution. What was regarded in Ref. 19 as enhanced stability (with the use of the median) of the test with the (5) statistic with respect to breakdown of the assumption of normal law behavior compared to distributions with "heavier" tails than a normal distribution, is, in fact, a change in $G(W|H_0)$ connected with the change in the method for estimating the mathematical expectation of the corresponding samples. How the distribution of the (5) statistic changes with the estimate that is used – the mean or the median – in the case of $m = 2$ samples of size $n = 10$ adhering to a normal law, is illustrated in Fig. 5. Also shown there is the corresponding $F_{1.18}$ Fisher distribution. Figure 5 shows that, for limited sample sizes, the difference in the distributions of the statistics owing to the use of different estimates must be taken into account.

TABLE 1. Power of Tests of the Homogeneity of Variances Relative to the Competing Hypothesis $H_1 : \sigma_2 = 1.1\sigma_1$

| Test | α | Sample size $n$ | | | | |
|---|---|---|---|---|---|---|
| | | 10 | 20 | 40 | 60 | 100 |
| Bartlett, Cochran, Hartley, Fisher | 0.1 | 0.112 | 0.127 | 0.158 | 0.188 | 0.246 |
| | 0.05 | 0.058 | 0.068 | 0.090 | 0.112 | 0.157 |
| | 0.01 | 0.012 | 0.016 | 0.024 | 0.032 | 0.051 |
| Levene | 0.1 | 0.110 | 0.123 | 0.150 | 0.176 | 0.228 |
| | 0.05 | 0.056 | 0.065 | 0.084 | 0.103 | 0.141 |
| | 0.01 | 0.012 | 0.014 | 0.021 | 0.028 | 0.044 |

TABLE 2. Power of Tests of the Homogeneity of Variances Relative to the Competing Hypothesis $H_2 : \sigma_2 = 1.2\sigma_1$

| Test | α | Sample size $n$ | | | | |
|---|---|---|---|---|---|---|
| | | 10 | 20 | 40 | 60 | 100 |
| Bartlett, Cochran, Hartley, Fisher | 0.1 | 0.144 | 0.199 | 0.304 | 0.401 | 0.564 |
| | 0.05 | 0.079 | 0.119 | 0.201 | 0.283 | 0.438 |
| | 0.01 | 0.018 | 0.033 | 0.071 | 0.114 | 0.218 |
| Levene | 0.1 | 0.135 | 0.184 | 0.276 | 0.363 | 0.515 |
| | 0.05 | 0.072 | 0.107 | 0.177 | 0.250 | 0.388 |
| | 0.01 | 0.016 | 0.028 | 0.058 | 0.095 | 0.180 |

**The Fisher test.** It is used for testing the hypothesis of equal variances for two samples of random quantities. The statistics for this test have the simple form

$$F = s_1^2 / s_2^2,$$

where $s_1^2$, $s_2^2$ are unbiased estimates of the variances calculated from the samples.

When the samples have normal distributions and $H_0 : \sigma_1^2 = \sigma_2^2$ is true, this statistic follows an $F_{\nu_1,\nu_2}$ Fisher distribution with $\nu_1 = n_1 - 1$, $\nu_2 = n_2 - 1$ degrees of freedom, where $n_1$ and $n_2$ are the sizes of the samples being compared. The hypothesis under test is rejected for small $F^* < F_{\alpha/2,\nu_1,\nu_2}$ or large $F^* > F_{1-\alpha/2,\nu_1,\nu_2}$ values of the statistic. As are the other tests, the Fisher test is very sensitive to deviations from normal law behavior.

**Comparative analysis of power.** The advantages of a particular test for a given probability α of a type I error (rejecting a true hypothesis $H_0$) can be judged in terms of the power $1 - \beta$, where β is the probability of a type II error (not rejecting hypothesis $H_0$ when the competing hypothesis $H_1$ is true). The lower power of the Cochran test compared to the Bartlett test is indicated clearly in the Bolshev–Smirnov tables [16]. For the example of a test of the homogeneity of the variances of five samples, it has been shown that the power of the Cochran test is higher [3].

For significance levels (probabilities of type I errors) $\alpha = 0.1$, 0.05, and 0.01, Tables 1–3 list the powers of these tests for the homogeneity of the variances for two samples ($H_0 : \sigma_2 = \sigma_1$) with respect the competing hypotheses $H_1 : \sigma_2 = 1.1\sigma_1$, $H_2 : \sigma_2 = 1.2\sigma_1$, $H_3 : \sigma_2 = 1.5\sigma_1$ (for normal law samples). The powers for the Bartlett, Cochran, Hartley, and Fisher tests all turned out to be the same in this case. When the powers were calculated from model distributions of the statistics (with true $H_0$ and $H_i$, $i = 1, 2, 3$), all three significant digits were the same in the tables. Thus, under these conditions (two samples,

TABLE 3. Power of Tests of the Homogeneity of Variances Relative to the Competing Hypothesis $H_3 : \sigma_2 = 1.5\sigma_1$

| Test | α | Sample size $n$ | | | | |
|---|---|---|---|---|---|---|
| | | 10 | 20 | 40 | 60 | 100 |
| Bartlett, Cochran, Hartley, Fisher | 0.1 | 0.312 | 0.532 | 0.806 | 0.926 | 0.991 |
| | 0.05 | 0.201 | 0.402 | 0.705 | 0.871 | 0.980 |
| | 0.01 | 0.064 | 0.182 | 0.463 | 0.692 | 0.924 |
| Levene | 0.1 | 0.269 | 0.471 | 0.746 | 0.888 | 0.981 |
| | 0.05 | 0.163 | 0.338 | 0.628 | 0.812 | 0.960 |
| | 0.01 | 0.045 | 0.131 | 0.364 | 0.590 | 0.866 |

TABLE 4. Power of Multisample Tests of the Homogeneity of the Variances with Respect to the Competing Hypothesis $H_1 : \sigma_m = 1.1\sigma_1$ for $n = 100$

| α | Tests | | | |
|---|---|---|---|---|
| | Cochran | Bartlett | Hartley | Levene |
| $m = 3$ | | | | |
| 0.1 | 0.250 | 0.242 | 0.239 | 0.225 |
| 0.05 | 0.161 | 0.152 | 0.148 | 0.139 |
| 0.01 | 0.056 | 0.049 | 0.046 | 0.043 |
| $m = 5$ | | | | |
| 0.1 | 0.241 | 0.224 | 0.219 | 0.209 |
| 0.05 | 0.156 | 0.138 | 0.133 | 0.127 |
| 0.01 | 0.056 | 0.044 | 0.040 | 0.039 |

TABLE 5. Power of Multisample Tests of the Homogeneity of the Variances with Respect to the Competing Hypothesis $H_2 : \sigma_m = 1.2\sigma_1$ for $n = 100$

| α | Tests | | | |
|---|---|---|---|---|
| | Cochran | Bartlett | Hartley | Levene |
| $m = 3$ | | | | |
| 0.1 | 0.609 | 0.577 | 0.568 | 0.530 |
| 0.05 | 0.494 | 0.459 | 0.443 | 0.409 |
| 0.01 | 0.286 | 0.237 | 0.217 | 0.200 |
| $m = 5$ | | | | |
| 0.1 | 0.624 | 0.557 | 0.545 | 0.513 |
| 0.05 | 0.515 | 0.434 | 0.418 | 0.390 |
| 0.01 | 0.316 | 0.227 | 0.204 | 0.197 |

TABLE 6. Power of Multisample Tests of the Homogeneity of the Variances with Respect to the Competing Hypothesis $H_3 : \sigma_m = 1.5\sigma_1$ for $n = 100$

| α | Tests | | | |
|---|---|---|---|---|
| | Cochran | Bartlett | Hartley | Levene |
| $m = 3$ | | | | |
| 0.1 | 0.997 | 0.996 | 0.995 | 0.990 |
| 0.05 | 0.994 | 0.990 | 0.988 | 0.979 |
| 0.01 | 0.974 | 0.962 | 0.947 | 0.926 |
| $m = 5$ | | | | |
| 0.1 | 0.998 | 0.996 | 0.995 | 0.991 |
| 0.05 | 0.997 | 0.991 | 0.989 | 0.982 |
| 0.01 | 0.987 | 0.969 | 0.955 | 0.944 |

extracted from normal-law ensembles) these four tests are equivalent. At the same time, the Bartlett test is to be preferred, since its asymptotic distribution, which is independent of the sample sizes (which may be unequal), is known. The Levene test clearly is inferior to the Bartlett, Cochran, Hartley, and Fisher tests in terms of power.

For non-normal-law distributions, such as when two samples to be analyzed adhere to the (1) family, the Bartlett, Cochran, Hartley, and Fisher tests are still equivalent in power, and the Levene test is noticeably inferior to them. However, for distributions with "heavier" tails (e.g., when the samples obey a Laplace distribution), the Levene test is better in terms of power.

It is necessary to know for what sizes of samples with specified probabilities of type I and type II errors, e.g., $\alpha \leq 0.1$ and $\beta \leq 0.1$, the Bartlett, Cochran, Hartley, and Fisher tests are able to distinguish the competing hypotheses $H_0$ and $H_i$. In the case of the relatively distant competing hypothesis $H_3 : \sigma_2 = 1.5\sigma_1$, for $n_1 = n_2$ the sample sizes $n_i$ must be at least 53, while in the case of the closer hypothesis $H_1 : \sigma_2 = 1.1\sigma_1$, on the order of 950 observations will be required.

The Bartlett, Cochran, Hartley, and Levene tests can be used for more than two samples. Even in those cases, the power of these tests turns out to be different. Tables 4–6 list the estimated powers (for normal-law samples) for $m = 3$ and 5 samples, in the case where the $(m - 1)$st sample corresponds to the value $\sigma_1$ and the latter to $\sigma_m$ unequal to $\sigma_1$.

Thus, in a multisample variant with normal law behavior, these tests can be ordered in terms of decreasing power as follows:

$$\text{Cochran} \succ \text{Bartlett} \succ \text{Hartley} \succ \text{Levene}.$$

This order of preference also holds when the assumption of normal-law behavior no longer holds. An exception is situations where the samples have distributions with "heavier" tails than a normal distribution, such as a Laplace distribution. Then the Levene test is somewhat more powerful than the other three.

**Conclusions.** This study of the distributions of the statistics and power of parametric tests of the homogeneity of variances shows the following: first, in an analysis of two samples the Bartlett, Cochran, Hartley, and Fisher tests have identical powers, both for normal-law samples and otherwise. Evidently, when the assumption of normal-law behavior holds, it is better to use the Bartlett test, since the distributions of its statistic are essentially independent of the sample sizes.

The Cochran test can be recommended for analyzing more than two samples. Although the literature contains rather limited data with percentile tables for the statistics of this test (for different numbers of samples and sample sizes), under modern conditions this lack can be overcome rapidly through the use of computer simulations [14].

If the assumption of normal-law samples does not hold, then the distributions of all these tests depend on the sample volumes (the Bartlett test loses its advantage). Naturally, the distributions of the statistics for the tests depend on the form of the distribution laws for the samples. It should be noted that the powers of all the tests examined here with respect to the same competing hypotheses are higher when the tails of the symmetric distributions of the samples analyzed here are "lighter" than in a normal distribution. Here, as a rule, the Cochran test is to be preferred. However, for distributions with "heavier" tails, the power of the Levene test is greater.

Later we shall examine the application and power of nonparametric tests used for testing hypotheses of the homogeneity of dispersion characteristics, and establish the feasibility of using the classical criteria when the quantities being analyzed are not normally distributed.

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