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Software System for Simulation and Research of Probabilistic Regularities and Statistical Data Analysis in Reliability and Quality Control

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Abstract: The computer approach to the investigation of estimation methods and statistical tests is considered as an effective technique for developing apparatus of applied mathematical statistics. It has been shown that basing on the considered approach and software system one can investigate statistical properties of estimates for distribution parameters including estimates by grouped and censored samples. The statistic distributions of nonparametric goodness-of-fit tests in testing composite hypotheses have been investigated. The statistic distributions and the power of χ^2 goodness-of-fit tests have been investigated depending on the number of intervals and the grouping method. A number of tests for deviation from the normal distribution law have been investigated. Homogeneity tests (for testing hypotheses about equality of means, equality of variances and homogeneity of distributions) have been studied. Various classical tests have been investigated in case of non-normal distributions of observations.

Keywords and phrases: Computer simulation, Nonparametric goodness-of-fit tests, χ^2 goodness-of-fit tests, Normality tests, Tests for homogeneity of distributions, Tests for homogeneity of means, Tests for homogeneity of variances, The test power

32.1 Introduction

The practice of using statistical analysis methods in applications is full of various problems whose statements are not described within the framework of classical assumptions. A wide range of statistical methods are based on the assumption of measurement error normality. Under real conditions normality and often some other assumptions are not satisfied. The use of classical methods of mathematical statistics in such situations can turn out to be incorrect.

Many classical results have an asymptotical nature. At the same time in practice one usually works with samples of a limited size. The application of asymptotical results is not always valid for limited sample sizes.

The form of data (measurements) registration doesn't often conform to complete samples considered in mathematical statistics textbooks. Actually, samples of

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observations can be grouped, censored, partially grouped or interval. Mathematical techniques must give an ability to analyze data in any form and must take into account this form and not to neglect it.

As a rule revealing fundamental statistic regularities in nonstandard conditions is a complicated problem for researchers. And the analytical methods for investigating properties of statistical estimates and test statistic distributions are very difficult and as a result of their complexity don't allow researchers to solve a great number of problems. The best way out is to use the numerical approach that is computer modeling of statistical regularities under conditions which simulate some real situations of measurement taking. Then mathematical models approximating the regularities obtained are constructed. Such an approach allows us to obtain good results in dealing with problems which are difficult to solve by analytical methods only. That is why computer simulation methods for statistical regularity analysis are becoming more and more popular.

This paper is devoted to the consideration of results obtained in various chapters of applied mathematical statistics with usage of the developed computer approach and software system meant for research of statistical regularities and statistical data analysis.

32.2 The Investigation of Parameter Estimates Properties

It has been shown in [Lem97a, Lem97b] that the usage of data grouping in tasks of distribution model identification enables to obtain robust estimates, eliminating an influence of gross measurement errors existed in samples. And the usage of asymptotically optimal grouping, for which losses of the Fisher information are minimized, enables to obtain estimates with good asymptotical properties.

The Fisher information losses caused by sample censoring have been considered in [LGP01, Lem01]. It has been shown that in some cases even for the considerable censoring degree the losses of the Fisher information induced with censoring of samples are not large. This enables to obtain rather good estimates of distribution parameters. The distributions of maximum likelihood estimates (MLE) of distribution parameters by censored samples have been investigated by computer simulation methods for various censoring degrees and various sample sizes. It has been shown that for the limited sample sizes the distribution of MLE turns out to be asymmetric and MLE is biased. The distribution laws frequently used in "life time" data analysis, such as lognormal, exponential, gamma, Rayleigh, Weibull, and other distributions have been considered.

32.3 The Investigation of Nonparametric Goodness-of-Fit Test Statistic Distributions

In composite hypotheses testing of the form $H_0 : F(x) \in \{F(x, \theta), \quad \theta \in \Theta\}$, when the estimate $\hat{\theta}$ of the scalar or vector distribution parameter $F(x, \theta)$ is calculated by

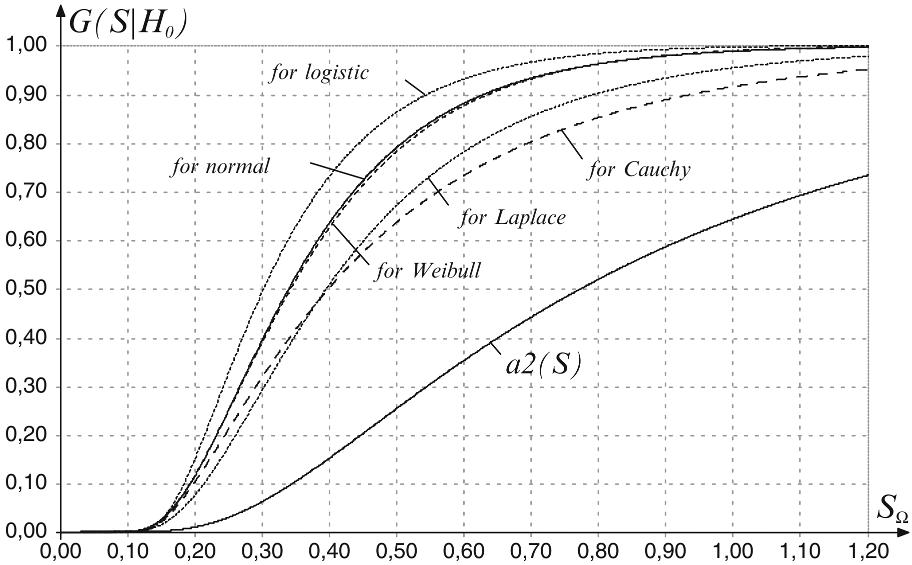

Figure 32.1. The Anderson–Darling statistic distributions for testing composite hypotheses with calculating MLE of two law parameters

the same sample, the nonparametric goodness-of-fit Kolmogorov, ω^2 Cramer–Mises–Smirnov, Ω^2 Anderson–Darling tests lose the free distribution property [KKW55]. In this case the conditional distribution law of the statistic $G(S\,|H_0\,)$ is affected by a number of factors: the form of the observed law $F(x, \theta)$ corresponding to the true hypothesis H_0; the type of the parameter estimated and the number of parameters to be estimated; sometimes, it is a specific value of the parameter (e.g., in the case of gamma-distribution and beta-distribution families and others); the method of parameter estimation [LP99]. The distinctions in the limiting distributions of the same statistics in testing simple and composite hypotheses are so significant that we cannot neglect them. For example, Fig. 32.1 shows distributions of the Anderson–Darling statistic while testing the composite hypotheses subject to various laws using maximum likelihood estimates (MLE) of two parameters.

Figure 32.2 illustrates the dependence of Kolmogorov test statistic distribution upon the type and the number of estimated parameters by the example of Su-Jonson law.

In our research [LP99, LP98a, LP01, Rec02ii, LM04a, Lem04, DELT04, LL09a, LNS09, LL09b, LL09c] statistic distributions of the nonparametric goodness-of-fit tests have been investigated by the methods of statistical simulating. Then basing on the obtained empirical statistic distributions we have constructed approximate analytical models of the statistic distribution laws. Table 32.1 contains a list of distributions relative to which we can test composite goodness-of-fit hypotheses using the constructed approximations of the limiting nonparametric statistic distributions.

One can use the models presented in [DELT04, LL09a, LNS09, LL09b, LL09c] in tasks of statistical data analysis, beginning from the sample size $n > 25$ and using the maximum likelihood estimates of unknown parameters.

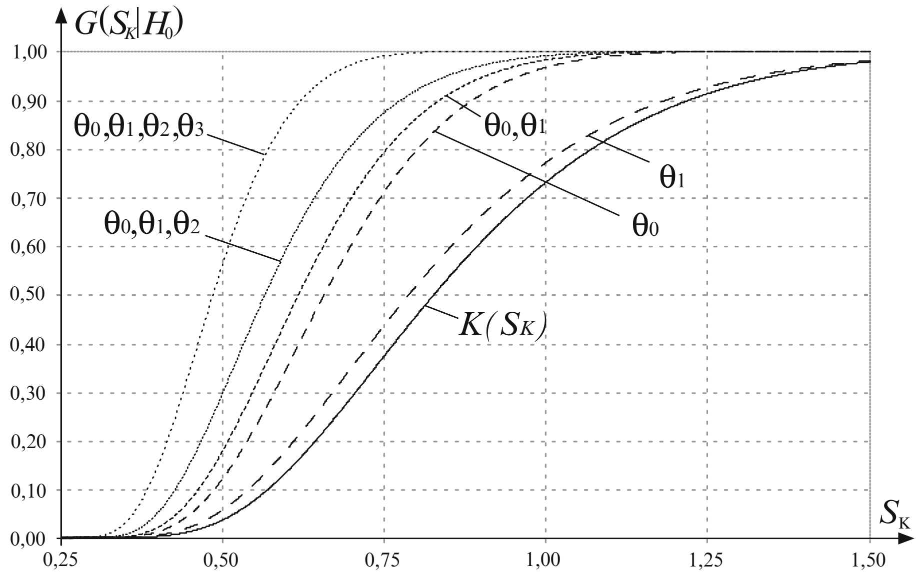

Figure 32.2. The Kolmogorov statistic distributions for testing composite hypotheses with calculating MLE of Su-Jonson distribution law parameters

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32.4 The Investigation of Statistic Distributions and the Power of χ^2 Tests

It has been shown in [DL79] that the less information losses caused by grouping are, the higher power of χ^2 tests (the Pearson χ^2 test and the likelihood ratio test) for close competing hypotheses.

Information losses can be decreased by selecting boundary points so, that $\mathbf{J}_G(\theta)$ tends to the information matrix for nongrouped data $\mathbf{J}(\theta)$, i.e. by solving asymptotically optimal grouping problem.

In case of scalar parameter, the problem reduces to the maximization of Fisher information quantity for grouped sample

$$\max_{x_{(1)}<x_{(2)}<\cdots<x_{(k-1)}} \sum_{i=1}^{k} \left(\frac{\partial \ln P_i\left(\theta\right)}{\partial \theta} \right)^2 P_i\left(\theta\right) = \max_{x_{(1)}<x_{(2)}<\cdots<x_{(k-1)}} J_G\left(\theta\right).$$

And in case of vector parameter various functionals of the Fisher information matrix can be chosen.

D-optim: the determinant of information matrix is maximized with respect to the boundary points

$$\max_{x_0<x_1<\ldots<x_{k-1}<x_k} \det \mathbf{J}_G(\theta)$$

(asymptotically D-optimal grouping problem).

Table 32.1. Random variable distribution laws

Random variable distribution	Density function $f(x,\theta)$	Random variable distribution	Density function $f(x,\theta)$
Exponential	$\frac{1}{\theta_0}e^{-x/\theta_0}$	Laplace	$\frac{1}{2\theta_0}e^{-\lvert x-\theta_1\rvert/\theta_0}$
Seminormal	$\frac{2}{\theta_0\sqrt{2\pi}}e^{-x^2/2\theta_0^2}$	Normal	$\frac{1}{\theta_0\sqrt{2\pi}}e^{-\frac{(x-\theta_1)^2}{2\theta_0^2}}$
Rayleigh	$\frac{x}{\theta_0^2}e^{-x^2/2\theta_0^2}$	Log-normal	$\frac{1}{x\theta_0\sqrt{2\pi}}e^{-(\ln x-\theta_1)^2/2\theta_0^2}$
Maxwell	$\frac{2x^2}{\theta_0^3\sqrt{2\pi}}e^{-x^2/2\theta_0^2}$	Cauchy	$\frac{\theta_0}{\pi[\theta_0^2+(x-\theta_1)^2]}$
Random variable distribution	**Density function $f(x,\theta)$**		
Logistic	$\frac{\pi}{\theta_0\sqrt{3}}\exp\left\{-\frac{\pi(x-\theta_1)}{\theta_0\sqrt{3}}\right\}\Big/\left[1+\exp\left\{-\frac{\pi(x-\theta_1)}{\theta_0\sqrt{3}}\right\}\right]^2$		
Extreme-value (maximum)	$\frac{1}{\theta_0}\exp\left\{-\frac{x-\theta_1}{\theta_0}-\exp\left(-\frac{x-\theta_1}{\theta_0}\right)\right\}$		
Extreme-value (minimum)	$\frac{1}{\theta_0}\exp\left\{\frac{x-\theta_1}{\theta_0}-\exp\left(\frac{x-\theta_1}{\theta_0}\right)\right\}$		
Weibull	$\frac{\theta_0 x^{\theta_0-1}}{\theta_1^{\theta_0}}\exp\left\{-\left(\frac{x}{\theta_1}\right)^{\theta_0}\right\}$		
Sb- Johnson $Sb(\theta_0,\theta_1,\theta_2,\theta_3)$	$\frac{\theta_1\theta_2}{(x-\theta_3)(\theta_2+\theta_3-x)}\exp\left\{-\frac{1}{2}\left[\theta_0-\theta_1\ln\frac{x-\theta_3}{\theta_2+\theta_3-x}\right]^2\right\}$		
Sl-Johnson $Sl(\theta_0,\theta_1,\theta_2,\theta_3)$	$\frac{\theta_1}{(x-\theta_3)\sqrt{2\pi}}\exp\left\{-\frac{1}{2}\left[\theta_0+\theta_1\ln\frac{x-\theta_3}{\theta_2}\right]^2\right\}$		
Su-Johnson $Su(\theta_0,\theta_1,\theta_2,\theta_3)$	$\frac{\theta_1}{\sqrt{2\pi}\sqrt{(x-\theta_3)^2+\theta_2^2}}$ $\times\exp\left\{-\frac{1}{2}\left[\theta_0+\theta_1\ln\left\{\frac{x-\theta_3}{\theta_2}+\sqrt{\left(\frac{x-\theta_3}{\theta_2}\right)^2+1}\right\}\right]^2\right\}$		
Gamma-distribution $\gamma(\theta_0,\theta_1,\theta_2)$	$\frac{1}{\theta_1^{\theta_0}\Gamma(\theta_0)}(x-\theta_2)^{\theta_0-1}e^{-(x-\theta_2)/\theta_1}$		
Double-exponential	$\frac{\theta_0}{2\theta_1\Gamma(1/\theta_0)}\exp\left\{-\left(\frac{\lvert x-\theta_2\rvert}{\theta_1}\right)^{\theta_0}\right\}$		
Beta-distribution of the I type	$\frac{1}{\theta_2 B(\theta_0,\theta_1)}\left(\frac{x}{\theta_2}\right)^{\theta_0-1}\left(1-\frac{x}{\theta_2}\right)^{\theta_1-1}$		
Beta-distribution of the II type	$\frac{1}{\theta_2 B(\theta_0,\theta_1)}\frac{[x/\theta_2]^{\theta_0-1}}{[1+x/\theta_2]^{\theta_0+\theta_1}}$		
Generalized Weibull	$\frac{\theta_0}{\theta_1}\theta_2^{\theta_0}x^{\theta_0-1}\left(1+\left(\frac{x}{\theta_2}\right)^{\theta_0}\right)^{\frac{1}{\theta_1}-1}e^{1-\left(1+\left(\frac{x}{\theta_2}\right)^{\theta_0}\right)^{\frac{1}{\theta_1}}}$		
Inverse Gaussian	$\left(\frac{\lambda}{2\pi x^3}\right)^{1/2}\exp\left(-\frac{\lambda(x-\mu)^2}{2\mu^2 x}\right)$		

A-optim: the trace of information matrix is maximized with respect to the boundary points

$$\max_{x_0<x_1<\cdots<x_{k-1}<x_k} Sp\mathbf{J}_G(\theta)$$

(asymptotically A-optimal grouping problem).

E-optim: the minimal eigenvalue of information matrix is maximized with respect to the boundary points

$$\max_{x_0<x_1<\cdots<x_{k-1}<x_k}\ \min_{i=\overline{1,r}}\lambda_i\left(\mathbf{J}_G(\theta)\right)$$

(asymptotically E-optimal grouping problem).

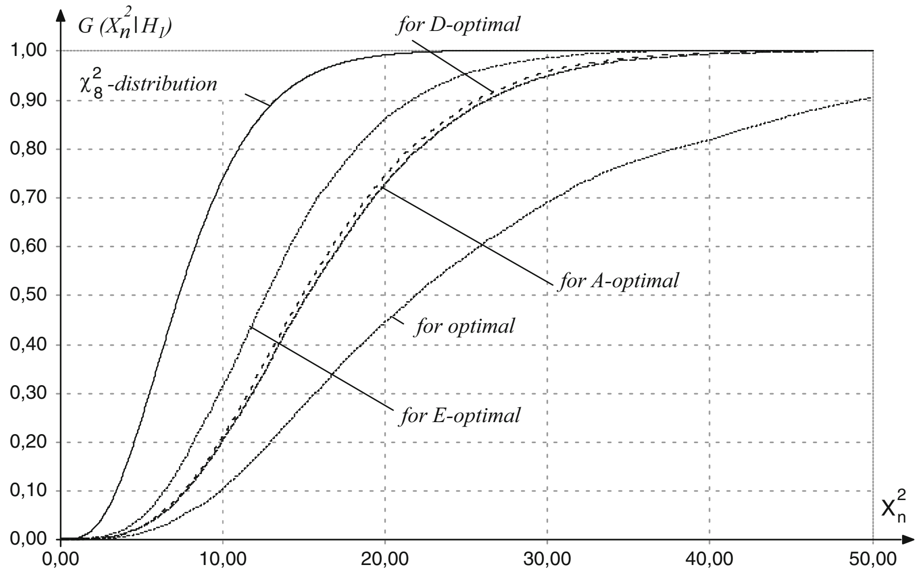

Figure 32.3. Distributions of Pearson's statistic X_n^2 in testing simple hypothesis H_0 if the hypothesis H_1 is true depending on the grouping method with $k = 9$ and $n = 500$

Optimum: the ν is maximized with respect to the boundary points

$$\max_{x_{(1)}<x_{(2)}<\cdots<x_{(k-1)}} s = \max_{x_{(1)}<x_{(2)}<\cdots<x_{(k-1)}} \left(n \sum_{i=1}^{k} \frac{\left(P_i(\theta_1) - P_i(\theta)\right)^2}{P_i(\theta)} \right).$$

The tables of asymptotically D-optimal grouping for rather wide range of distribution laws which are most frequently used in applications were constructed previously [DLT93]. At the present time we have solved the problems of A- and E-optimal grouping for a number of distributions. The tables of asymptotically optimal grouping which can be used in estimating distribution parameters by grouped samples and in testing goodness-of-fit have been constructed. The use of asymptotically optimal grouping tables ensures the maximal power of χ^2 tests for close competing hypotheses. In Fig. 32.3, there are the Pearson χ^2 test statistic distributions in testing simple hypothesis of goodness-of-fit to the normal distribution $H_0 : f(x) = \frac{1}{\theta_0\sqrt{2\pi}} \exp\left\{-\frac{(x-\theta_1)^2}{2\theta_0^2}\right\}, \theta_0 = 1, \theta_1 = 0$, in case of the true competing hypothesis $H_1 : f(x) = \frac{\pi}{\theta_0\sqrt{3}} \exp\left\{-\frac{\pi(x-\theta_1)}{\theta_0\sqrt{3}}\right\} \Big/ \left[1 + \exp\left\{-\frac{\pi(x-\theta_1)}{\theta_0\sqrt{3}}\right\}\right]^2, \theta_0 = 1, \theta_1 = 0$ (logistic distribution), in dependence on the grouping method.

It has been shown for the first time that there is an optimal number of intervals k depending on sample size, concrete alternatives and a way of grouping. The optimal number of intervals k depends on the sample size n and on the concrete pair of competing hypotheses H_0 and H_1. As a rule, the optimal k turns out to be significantly less than values recommended by a number of empirical formulas for the choice of k. In Fig. 32.4 the power functions of the Pearson χ^2 tests are represented depending on the interval number k in case of D-, A-, and E-optimal grouping in simple hypothesis testing, for $n = 200$ (H_0 : normal distribution; against H_1 : logistic distribution).

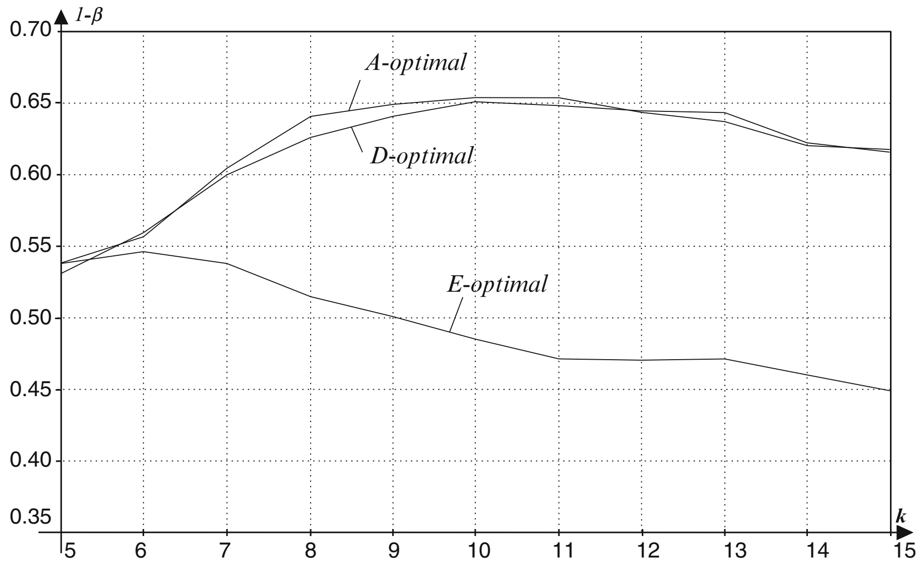

Figure 32.4. The dependence of the Pearson χ^2 test power on the number of intervals k for various grouping methods, $n = 200$, in case of testing H_0: normal distribution against H_1: logistic distribution

The results of investigating [Lem97c, Lem98, LP98b, LC00, LPC01, LC03, LC02] properties of the χ^2 goodness-of-fit tests (Pearson, Rao-Robson-Nikulin [Nik73, NC73, RR74, GN96]) were included to the developed recommendations [Rec02i].

The power of the χ^2 Dzhaparidze–Nikulin test has been investigated depending on the grouping method and the number of intervals. The problem of power maximization for the χ^2 Pearson and Rao–Robson–Nikulin tests has been investigated for specified pairs of competing hypotheses. Moreover, we have considered the use of the so called Neumann–Pearson intervals [GN96], for which the boundary points coincide with cross points of density functions of competing hypotheses. It has been shown that such intervals are reasonable to be used. But at the same time, the use of these intervals doesn't ensure the maximal power of the test for given pair of competing hypotheses.

32.5 The Comparative Analysis of the Power of Goodness-of-Fit Tests

The power of a number of nonparametric and parametric goodness-of-fit tests with respect to a series of pairs of competing hypotheses has been studied [LLP07, LLP09, LLP08] in case of testing simple and composite hypotheses by statistical simulation methods. We can rank the tests by power for simple hypothesis testing as follows:

$$\chi^2 \text{ Pearson (AOG)} \succ \Omega^2 \text{ Anderson–Darling} \succ \omega^2 \text{ Mises} \succ= \text{Kolmogorov}.$$

This scale holds while using asymptotically optimal grouping in the Pearson χ^2 test, which minimizes the losses in the Fisher information. For quite close hypotheses, the advantage by power of the Pearson χ^2 test can be essential.

In testing composite hypotheses the preference sequence turns out to be quite different:

$$\Omega^2 \text{ Anderson–Darling} \succ \omega^2 \text{ Mises} \succ Y_n^2 \text{ Rao–Robson–Nikulin (AOG)} \succ \\ \succ \chi^2 \text{ Pearson (AOG)} \succ \text{Kolmogorov.}$$

For very close competing hypotheses the following sequence can take place:

$$\Omega^2 \text{ Anderson–Darling} \succ Y_n^2 \text{ Rao–Robson–Nikulin (AOG)} \succ \omega^2 \text{ Mises} \succ \\ \succ \chi^2 \text{ Pearson (AOG)} \succ \text{Kolmogorov.}$$

The conclusions stated have an integrated nature.

32.6 The Investigation of Statistic Distributions and the Power of Normality Tests

Statistic distributions and the power of a number of criteria for testing deviation from the normal law (Shapiro-Wilk, Epps-Pulley, D'Agostino, Frosini, Hegazy-Green, Spiegelhalter, Geary, David-Hartley-Pearson and some others) have been investigated in [LL05a, LR09].

The considered tests can be ranked by power as follows:

$$\text{Geary} \succ \text{Spiegelhalter} \succ \text{Hegazy-Green } (T_2) \succ \text{Hegazy-Green } (T_1) \succ \\ \succ \text{Epps-Pulley} \succ \text{David-Hartley-Pearson} \succ \text{Shapiro-Wilk} \succ \text{Frosini.}$$

The advantages and disadvantages of various tests have been shown. It has been shown for the first time that for small sample sizes a number of tests are biased, including the Spiegelhalter, Shapiro-Wilk, Epps-Pulley and Hegazy-Green tests relative to symmetrical alternatives with the kurtosis value less than three. Some of the considered tests are not reasonable to be applied at all because of their fundamental disadvantages. The normality tests have been compared by power with the goodness-of-fit tests.

32.7 The Investigation of Homogeneity Test Statistic Distributions

The homogeneity tests are intended for checking whether two random samples represented by the variation series

$$x_1 < x_2 < \cdots < x_m \text{ and } y_1 < y_2 < \cdots < y_n$$

belong to the same distribution, i.e. $H_0 : F(x) = G(x)$ for any x.

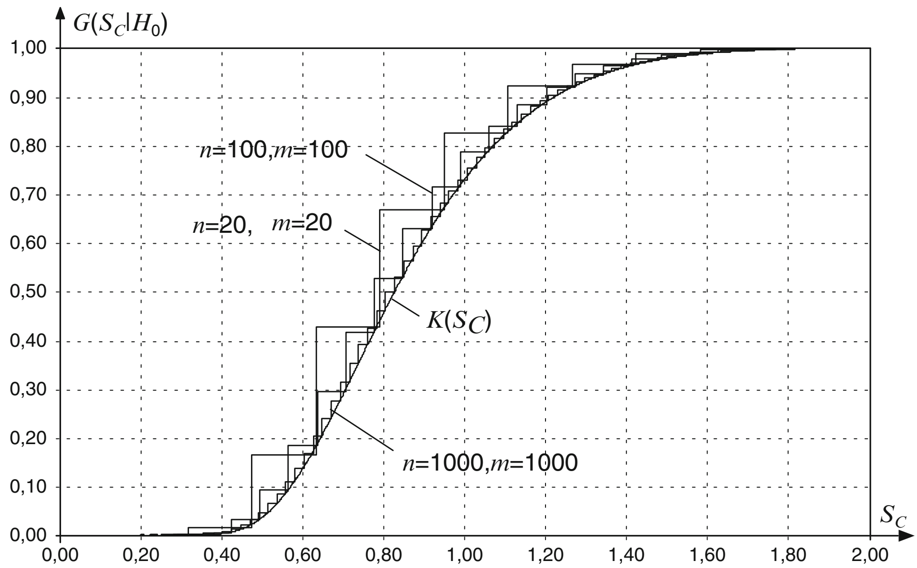

Figure 32.5. The Smirnov statistic distributions when the null hypothesis is true depending on the sample sizes m and n

The statistic distributions and the power of the Smirnov and Lehmann–Rosenblatt tests for homogeneity of two samples were investigated in [LL05b]. The Smirnov test statistic is a discrete random variable and its distribution converges slowly (from the left!) to the limiting Kolmogorov distribution (see Fig. 32.5). Hereupon, the use of the Kolmogorov distribution $K(s)$ as the limiting law when sample sizes are limited lead to the overrated values of significance level achieved and, hence, to increasing the number of beta errors. The recommendations of choosing sample sizes m and n are given in this paper. The empirical correction for the Smirnov statistic which improves convergence of the statistic distribution to the limiting Kolmogorov law has been obtained.

The power of the Lehmann–Rosenblatt test, as a rule, turns out to be higher than the power of the Smirnov test.

32.8 The Investigation of Statistic Distributions and the Power of Tests for Homogeneity of Means

The comparative analysis of the power of parametric and nonparametric criteria used for testing homogeneity of means has been carried out in [LL08]. In the general case, the hypothesis of mathematical expectations equality corresponding to k samples has the form

$$H_0 : \mu_1 = \mu_2 = \cdots = \mu_k$$

under the competing hypothesis

$$H_1 : \mu_{i_1} \neq \mu_{i_2}$$

for at least some pair of indices i_1, i_2.

There are a number of parametric tests that may be used to compare two sample means to check some hypothesis H_0: with known variances; with unknown, but equal variances (Student's test); with unknown and unequal variances; and with the F-test. There also exists a number of nonparametric tests that may be used for this purpose, e.g., the Wilcoxon, Mann–Whitney, and Kruskal–Wallis tests. Membership of the particular sample being analyzed to a normal law is the basic assumption determining whether parametric tests should be used. Nonparametric tests are free of this requirement.

It has been shown that parametric tests associated with testing a hypothesis of mathematical expectations are robust with respect to deviations of the observed laws from the normal distribution. If the distribution law (laws) of analyzed samples is different from the normal law but doesn't have the "heavy tails" than the use of parametric tests is correct, at least it doesn't result in considerable errors.

Some conclusions can be arrived on the basis of the test power investigation results. Firstly, the parametric tests have the greater power than do nonparametric tests. Secondly, it may be stated that nonparametric tests are absolutely slightly inferior in terms of power to parametric tests, thus, the Mann–Whitney test is inferior to the Student's test, and the Kruskal–Wallis test to the Fisher test, respectively.

32.9 The Investigation of Statistic Distributions and the Power of Tests for Homogeneity of Variances

One of the main assumptions which should be taken into account while constructing the classical tests for homogeneity of variances is the normality of observed random variables (measurement errors). Therefore, the application of classical tests is always associated with the question whether obtained conclusions are correct in a certain situation. The conditional test statistic distributions relative to a true hypothesis under test, as a rule, change significantly if the assumption of the normal distribution of analyzed random variables is disturbed.

The tested hypothesis about equality of variances in k samples has the form

$$H_0 : \sigma_1^2 = \sigma_2^2 = \cdots = \sigma_k^2,$$

And the competing hypothesis is

$$H_1 : \sigma_{i_1}^2 \neq \sigma_{i_2}^2,$$

where inequality holds at least for one pair of indices i_1, i_2.

The distributions of the classical test statistics have been investigated in case when distributions of observed random variables differ from the normal law. The possibility of the classical tests application under conditions of non-normal distributions has been studied. The comparative analysis by power of the classical variance homogeneity tests (Fisher's, Bartlett's, Cochran's, Hartley's and Levene's tests) and the nonparametric (rank) tests Ansari–Bradley, Mood's, Siegel–Tukey tests) have been carried out by statistical simulation methods in [LM04b, LP06] and further works.

We have investigated the power of Fisher's, Bartlett's, Cochran's, Hartley's and Levene's tests relative to the competing hypothesis of the kind $H_1 : \sigma_2 = d\sigma_1, d \neq 1$, for the number of samples $k = 2$ in case of normal distribution of random variables. It has been shown that in this situation the Fisher, Bartlett, Cochran and Hartley tests are equal by power. The Levene test considerably yields to them.

The Fisher, Bartlett, Cochran and Hartley tests remain to be equal by power if the random variable distribution is different from the normal law, e.g. in case of belonging of two analyzed samples to the double-exponential distribution law, and the Levene test yields to them. But in case of distributions with "heavy tails" the Levene test has an advantage in power.

The tests of Bartlett, Cochran, Hartley and Levene may be applied for number of samples $k > 2$. In such situations, the power of these tests turns out to be different. When the assumption of the normal distribution holds for $k > 2$ these tests may be ranked by power decrease as follows:

$$\text{Cochran} \succ \text{Bartlett} \succ \text{Hartley} \succ \text{Levene}.$$

This preference order also holds in case when the normality assumption is disturbed. An exception concerns the situations when samples belong to some distributions which have more heavy tails than the normal law. For example, in case of belonging samples to the Laplace distribution the Levene test turns out to be slightly more powerful than three others.

The results of investigating the nonparametric tests have shown an evident advantage of the Mood test and practical equivalence of the Ansari–Bradley and Siegel–Tukey tests. The nonparametric tests are obviously inferior by power to the Bartlett, Cochran, Hartley and Levene tests. In Fig. 32.6, there are the graphs of the test power relative to the competing hypotheses $H_1^1 : \ \sigma_2 = 1.1\sigma_1$ and $H_1^2 : \ \sigma_2 = 1.5\sigma_1$ depending on the sample sizes n_i for $\alpha = 0.1$ in case of the normal law. As it is seen from the figure the power of Cochran's test comparing with the most powerful nonparametric Mood's test is rather considerable. Let us remind that in case of two samples the power of the Fisher, Bartlett, Cochran and Hartley tests coincide.

The distributions of nonparametric tests don't depend on the observed distribution law if both samples belong to one and the same family of distributions. But if samples have different distributions than for the true tested hypothesis H_0 of variance homogeneity the distributions of nonparametric test statistics change: they depend on these distribution laws.

The classical tests have a considerable advantage by power over the nonparametric tests. This advantage remains even when analyzed samples belong to the distribution which is considerably different from the normal law. So there are all reasons for investigation of the classical test statistic distributions (construction of the distribution models or the tables of percentage points) for the non-normal laws frequently used in practice. The Cochran test is the most appropriate for this role among the considered tests.

The table of upper percentage points (1%, 5%, 10%) of the Cochran test (for the numbers of samples $k = 2/5$) have been constructed in case of some certain families

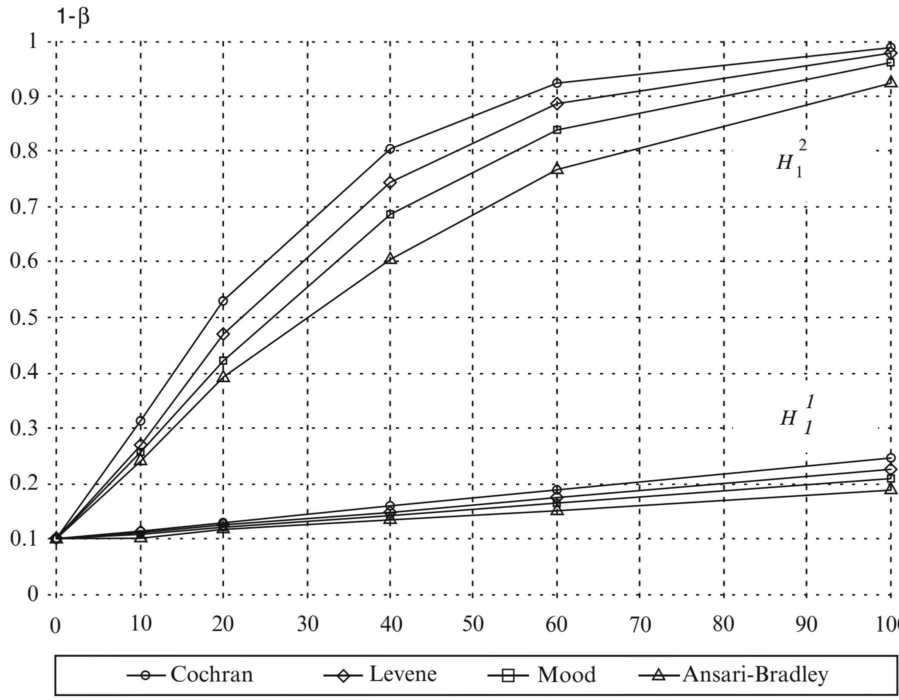

Figure 32.6. The power of tests relative to the competing hypotheses H_1^1 and H_1^2 depending on the sample size n for $\alpha = 0.1$ in case of the normal law

of observed random variable distributions for a number of sample size values n. The developed software system enables to solve this problem for any random variable distribution law and for any classical test of variance homogeneity, as well as it enables to construct models of statistic distributions for these tests when necessary.

32.10 Conclusion

The computer simulation technique of data analysis and investigation of probabilistic regularities have been used for solving other problems of applied mathematical statistics. In particular, we have investigated the robustness and power of the Abbe test used for testing hypothesis about the trend absence [Lem06]. The distributions of the Grabbs test statistic used in tasks of rejecting anomalous measurements have been investigated by statistical simulation methods in case when an observed law is different from the normal distribution [LL05c]. The statistic distributions of classical criteria of testing hypotheses about variances have been investigated when a random variable distribution differs from the normal law [LP04a]. We have developed the facilities of modeling and investigating the distribution laws of arbitrary functions of random variables and functions of random variable systems as well as the facilities of constructing approximate models for these distribution laws [LC07]. We have also developed the technique for simulation and investigation of distributions of multivariate random variables statistics [LP02].

So we can state that the computer technologies of data analysis and investigation of probabilistic and statistical regularities present the powerful tool for the development and improvement of the applied mathematical statistics apparatus including solving problems of reliability and survival analysis.

At the same time there are some own problems on the way of developing the computer technologies of data analysis and statistical regularities research [LP04b]. First of all, the construction of sufficiently precise models; for example, the models of test statistic distributions, basing on statistical simulation results frequently requires large amount of simulations (tens and hundreds hours of processor time). Secondly, the classical results involve mainly the most elementary situations. In more complicated cases, the decision can turn out to be ambiguous. For example, the distribution of some test statistic may depend on the value of a certain parameter of observed distribution law and cannot be expressed in the form of regularity, someway depending on this parameter. This means that the statistic distribution changes in dependence on solved task conditions. There exists the infinite number of combinations of the conditions and so it's impossible to construct an infinite number of models for all the affairs. Consequently, it turns out to be reasonable to construct a probabilistic model "in real time" when there occurs the necessity of decision making under conditions of available assumptions. It means that when testing a statistical hypothesis we have to specify the test statistic distribution, corresponding to the true hypothesis under test, in process of statistical analysis itself. And then basing on this distribution one will reject or will not reject the hypothesis under test. For the present condition of computing facilities and encouraging perspectives of their development the achievement of the purpose is feasible by organizing distributed computations using free facilities of computers and computer clusters in the networks. Our computing experiments have confirmed the possibility and efficiency of such approach because of comparatively simple paralleling simulation operations.

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